Abstract—A simple, yet popular way to design energy-efficient routing for Mobile Ad Hoc Networks (MANET) is to use a virtual backbone that forms a minimum sized Connected Dominating Set (CDS) of the network topology. By minimising the number of forwarding nodes, it looks at extending the (battery-dependent) life-span of the network by minimising the number (and energy cost) of transmitting nodes.

In this paper, we consider a more realistic model in which the energy cost of the receiving nodes (including nodes overhearing packets) is also taken into account, and show that current CDS algorithms may lead to backbones that are ineffective at minimising the overall energy cost during broadcast.

We first prove that a (realistic) reception-aware model leads to a new NP-complete problem - we have coined this Connected Exact Cover - to reduce the energy drain due to the number of overheard receptions while broadcasting in MANET. This holds even if all nodes transmit at the same power. Then we introduce two algorithms, one centralised and one distributed, and show with several simulations that these algorithms generate virtual backbones that consume less energy during broadcasts compared to the best virtual backbone schemes known in the literature.

I. INTRODUCTION

In this paper, we are concerned with the energy consumed by Mobile Ad Hoc Network (MANET) protocols as a performance metric. We are interested in characterising some of the energy-efficient features required by MANET protocols to route with minimum energy cost and extend the life of the energy source of each mobile node. As some devices have the ability to adjust power, most of the theoretical work has led to many NP-complete problems (e.g., energy-optimal broadcast [4], [7], broadcasting increment power heuristics [3], smallest common power level which results in connectivity of the network [15], energy-dependent participation based on the remaining level of their battery [17]).

These studies have focused on transmitting energy as the sole energy cost. However, as pointed out in Feeney and Nilsson’s seminal work [10] regarding the energy consumption of MANET current interfaces, there are other substantial costs to consider. In MANET, there is no infrastructure and the nodes cannot go into an energy-saving (“sleep”) mode easily as the intrinsic ad hoc nature of the network forces them to remain in a “ready to receive” state (i.e., the so-called “idle” state). Overhearing any packet is costly because the conservation of energy occurs only after discerning that an incoming packet is not intended for a node. For instance, in [10] the Lucent IEEE 802.11 2Mbps WaveLAN PC Card power consumption characteristics were measured as: idle power at 843mW; receive power at 967mW; transmit power at 1327mW. There is a large body of work targeted to substantially reduce this “idle state” cost and the difference of cost between the reception and idle states is expected to widen (e.g., [6]).

However the tenet that the reception cost is larger than a substantial fraction of the transmission cost remains. Let us define the energy costs of the idle state, the transmission interface and the reception interface for a node $i$ as $\tau_i$, $\tau_i$ and $\rho_i$, respectively. We can define the energy net costs for the transmission interface and the reception interface for a node $i$ as $t_i(=\tau_i-\tau_i)$ and $r_i(=\rho_i-\tau_i)$, respectively. The passive receptions cumulated from non-targeted neighbours may cost more than the transmission cost from the source and the reception cost from the targeted neighbours. With the Lucent specifications above, the net energy costs are $t_i=484$mW and $r_i=124$mW respectively, where a network with an average degree of five neighbours will consume more energy through receptions than through transmissions. This leads to a detrimental drain of neighbouring nodes with low battery even if they did not need to be involved in the transmission. Here, we study the impact of link interference on energy efficient routing in MANET by considering the energy cost of transmitting and receiving packets (even when all nodes transmit at the same power). As overall consumption is dependent of the mobile hardware, we restrict our study to considering “unit” transmission and reception energy costs of the wireless network interface. In the rest of the paper, we will assume that $t_i$ and $r_i$ are fixed at each node $i$.

For clarity, and without loss of generality, we introduce a simple graph model (which does not impact on our complexity results). (For basic graph-theoretical definitions, see Diesel [9].) We consider an undirected graph $G(V,E)$ to model a wireless network. Let $N(i)$ be the neighbours of node $i$. Denote by $N_i(h)$ the set of nodes that are at most $h$ hops away from node $i$ (including node $i$), e.g., $N_i(1) = \{i\} \cup N(i)$. A transmission interferes with nodes that are within $H_i$ hops, from the transmitter, depending on the signal to noise ratio.
When node \( i \) transmits a packet to its neighbour \( j \), all nodes in \( N_i(H_1) \) can receive the packet and may need to process it. Consuming \( t_i \) units of interface energy (at node \( i \)) for transmission from \( i \) to node \( j \) induces the consumption of \( r_k \) units of interface energy at each receiving node \( k \) within \( H_1 \) hops from \( i \) (including \( j \)). In this context, it is easy to compute the worst-case per-packet cost for passing a packet along a path \( \Pi \) from a source \( s \) to a destination \( d \):

\[
\sum_{i \in \Pi \setminus \{d\}} (t_i + \sum_{j \in N_i(H_1) \setminus \{i\}} r_j)
\]

Let \( K_\Pi(j) = \{i \in \Pi \setminus \{d\} | j \in N_i(H_1) \setminus \{i\}\} \), that is, the nodes on the path \( \Pi \) that are within \( H_1 \) hops from a node \( i \) of the path \( \Pi \). Let \( k_\Pi(j) = |K_\Pi(j)| \). The per-node cost at a node \( j \) (for passing a packet along a path \( \Pi \) from a source \( s \) to a destination \( d \), possibly not including \( j \)) is defined as:

\[
\begin{align*}
& r_j \text{ for } j \in \Pi, \\
& t_j + \sum_{i \in K_\Pi(j)} r_j \text{ for } j \notin \Pi,
\end{align*}
\]

The second case formalizes the fact that the battery of a node in the vicinity of some transmissions may be drained rapidly without transmitting once. This also emphasises the fact that the per-packet cost and per-node cost lead to two distinct problems, even when all the nodes transmit at the same energy cost. It is known that minimising the per-node cost is NP-complete if some minimum remaining energy is required [1].

In Section II, we first prove that a (realistic) reception-aware model leads to a new NP-complete problem - we have coined this Connected Exact Cover - to reduce the energy drain due to the number of overheard receptions while broadcasting in MANET (even if all nodes transmit at the same power). In Section III we introduce two algorithms, one centralised and one distributed, to reduce the energy drain due to the number of overheard receptions in MANET. We show with several simulations that our algorithms generate virtual backbones that consume less energy during broadcast compared to the best virtual backbone schemes known in the literature.

II. RECEPTION-AWARE BROADCAST AND VIRTUAL BACKBONE

Numerous hierarchical topology structures have been suggested in the literature. Almost all schemes lead to a well-known combinatorial problem called the Minimum Dominating Set (DS) problem that is NP-complete [11]. This scheme can be refined to Connected Dominating Set (CDS) to allow straightforward routes between different dominators or cluster heads, and hence form a connected induced component (i.e., a virtual backbone). Again, the size of the backbone must be minimum and these problems remain NP-complete.

The NP-complete Minimum Connected Dominating Set problem [11] is defined as follows.

**Definition 1:** Minimum Connected Dominating Set (MCDS) is defined as:

**Instance:** A Graph \( G = (V, E) \) and an integer \( B \).

Define a **Connected Dominating Set (CDS)** \( C \) of the graph \( G \) a subset of \( V(G) \) such that:

- for every vertex \( v \in V(G) \), either \( v \in C \) or there exists a vertex \( u \in C \) that is adjacent to \( v \) in \( G \).
- the subgraph induced by the vertices of \( C \) in \( G \) is connected.

**Question:** Is there a CDS \( C \) for \( G \) such that \( |C| \leq B \)?

When the number of receptions is considered (to minimise the energy cost), one would like to obtain a virtual backbone with as few as possible cover sets but these should cover disjoint sets, if possible. Informally, the goal is to build a CDS of small size where non-dominating nodes have as few as possible dominating neighbours.

One approach could be to introduce a weighted version of a CDS heuristic that takes into account the number of triggered receptions by a transmission at a node. Each node \( i \) can locally compute the maximal possible reception energy cost by summing the cost of all its neighbours and setting its weight \( w_i \) to \( t_i + \sum_{j \in N_i(H_1) \setminus \{i\}} r_j \). By taking the energy-cost as the metric (on a per node-basis), an algorithm using weights can be easily designed and will obtain the same approximation performance [13] (although it is independent of the weight function chosen). However, this does not directly reduce the number of unnecessary receptions at any individual node, since a good solution for the overall network may be fatal for several nodes which will be drained of their energy even if they do not communicate. Hence another possibility is to modify the combinatorial problem objective altogether by minimising the number of dominating overlaps by dominators. More importantly, as the weights are specific to the graph, we must first assess if this particular problem can be solved optimally in polynomial time.

We now introduce the new problem of a connected backbone with reception-awareness (i.e., a CDS of small size where non-dominating nodes have as few as possible dominating neighbours) and define it formally as follows. (As it seems to be unknown in the literature, we have coined it the Exact-Cover CDS or Connected Exact Cover problem, for short.)

**Definition 2:** Connected Exact Cover (CEC) is defined as:

**Instance:** A Graph \( G = (V, E) \) and an integer \( B' \).

**Question:** Is there a Connected Dominating Set \( C \) for \( G \) such that \( \sum_{u \in C} |N(u)| \leq B' \)?

As the only difference with the MCDS problem is the metric used for the optimisation question, we must first prove that unfortunately it remains NP-complete.

**Theorem 3:** The Connected Exact Cover problem (CEC) is NP-complete.

Our proof follows a polynomial reduction from this problem to the minimum Exact Set Cover problem (EC) which is NP-complete [13] and is defined as follows.

**Definition 4:** Minimum Exact Cover (EC) is defined as:

**Instance:** A Collection \( S \) of \( m \) subsets \( \{S_1, S_2, \cdots, S_m\} \) of a finite set \( U \) and an integer \( B \).

**Question:** Is there an exact cover for \( U \), i.e., a subset \( S' \subseteq S \) such that every element in \( U \) belongs to at least one member...
of $S^*$, such that $\sum_{S_i \in S^*} |S_i| \leq B$?

Proof: It is easy to see that $CEC \in NP$, because a nondeterministic algorithm needs only to guess a set $C$ and to check in polynomial time that it is a connected dominating set and the constraint $\sum_{v \in C} |N(v)| \leq B'$ is valid.

We now use a polynomial reduction from this problem to the minimum Exact Set Cover problem (EC). Given an Exact Set Cover instance (using the notation of the definition above), construct a graph $G(V, E)$ such that:

- $V = \{u, v, v_1, v_2, \ldots, v_m\} \cup U$,
- $E = E_U \cup E_S \cup \{(u, v)\}$, where $E_U$ contains an edge for each element of $U$ to the corresponding representative node $v_i$ of the set $S_i$, and $E_S$ contains an edge from each node $v_i$ to the node $v$.

As $u$ is only connected to $v$, a minimum Connected Exact Cover $C$ on the graph $G(V, E)$ will have to contain $v$ (and will not contain $u$ by minimality). Similarly, no node representing an element of $U$ will belong to $C$ (by connectedness, one of its neighbors $v_i$, representing a set $S_i$ is already included in $C$). Hence $C$ is only composed of $v$ and a minimum set of nodes $v_i$, that must correspond exactly to a minimum exact cover of the given instance of the given Exact Set Cover.

As the proof follows a polynomial reduction to the $EC$ problem (since it is easy to check that the graph construction is polynomial), the $CEC$ problem is NP-complete.

As the size of the solution set in the reduction is exactly one more the size of the given Exact Set Cover problem, we also deduce the following corollary.

**Corollary 5**: The Connected Exact Cover problem ($CEC$) is approximable within $1 + \ln |V|$ [13] but as hard to approximate as Set Cover [14]. When the number of neighbours of each node of $V$ is bounded by a constant $\Delta$ independent of the size of the input, it is approximable within $H(\Delta + 1) = \sum_{j=1}^{\Delta + 1} \frac{1}{j}$ (the Harmonic function) [13]. This ratio is slightly less than $1 + \ln(\Delta + 1)$ (as $H(\Delta + 1) < 1 + \ln(\Delta + 1) < H(\Delta + 1) + \frac{1}{\Delta}$).

### III. CEC HEURISTICS

In order to address the NP-completeness of the Connected Exact Cover problem, we propose two heuristics to minimise the number of (transmissions and) receptions induced by the routing protocol by adequately adapting two commonly used schemes. Our simulations will also show that, although the only difference between $CDS$ and $CEC$ is the definition of the objective function, the two problems are likely to diverge showing that our proposal is not a slight formal modification.

#### A. Centralised Heuristics

Initially in our model, we consider that each node knows the topology of the ad hoc network. This can be easily carried out if we suppose that the underlying routing protocol works on a proactive basis. As proactive routing protocols use link state information, nodes actually have knowledge of the topology of the network (e.g., OLSR [8]).

The proposed algorithm is a modification of the centralised algorithm in [12] that is known to produce a CDS with the lowest approximation factor for arbitrary graphs. The original algorithm has an inclusion step that aims to maximise the coverage of uncovered nodes without consideration of the nodes that have already been covered by the set.

In the proposed algorithm, the inclusion step is modified to maximise the coverage of uncovered nodes while minimising the coverage of nodes already covered by the set. This new coverage scheme is denoted as an exact coverage (i.e., ideally, each node is either a dominator or is covered exactly once). Thus, rather than selecting nodes that produce the maximum cover, the proposed algorithm selects nodes that produce the lowest exact yield (defined below).

The following is a pseudocode of the proposed algorithm.

1. **Initialise** all nodes white.
2. Colour the node with the highest degree black and its neighbours grey.
3. **WHILE** there exist white nodes:
   1. Select either the grey node, or the grey node and its white neighbours, that produces the lowest exact yield.
   2. Colour the selected node(s) black and their white neighbours grey.

The exact yield of a single grey node $u$ is calculated by the following ratio:

$$\text{yield}_{\text{exact}}(u) = \frac{|N_{\text{grey}}(u)|}{|N_{\text{white}}(u)|}$$ (3)

The exact yield for a pair of adjacent nodes $u$ and $v$, where $u$ is a grey node and $v$ is a white node, is defined as follows:

$$\text{yield}_{\text{exact}}(u, v) = \frac{|N_{\text{grey}}(u) \cup N_{\text{grey}}(v) \setminus (u \cup v)| + |N_{\text{white}}(u) \cap N_{\text{white}}(v)|}{|N_{\text{white}}(u) \cup (u \cup N_{\text{white}}(v))|}$$ (4)

The numerator in the exact yield ratio is the number of times that node $u$ and node $v$ will increase the overlap in nodes that have already been covered by another dominator, combined with the number of nodes that will experience overlap due to being covered by both node $u$ and node $v$. The denominator is the total number of uncovered nodes that will be covered if node $u$ and node $v$ become dominators.

The exact yield is calculated for every grey node $u$ and for every pair of adjacent nodes $u$ and $v$ where $u$ is grey and $v$ is white. The node or pair of nodes that produces the smallest exact yield are added to the CDS. More formally, this calculation is described as finding either a single node $u$ or a node pair $u$ and $v$ that satisfy $\min_{u \in \text{grey, } v \in \text{white}} |N_{\text{u}}(u) \cap N_{\text{v}}(v)| / |N_{\text{white}}(u) \cap (u \cup N_{\text{white}}(v))|$.
B. CEC Distributed Heuristics

The most efficient CDS distributed algorithms (e.g., [2], [5]) use the concept of an Independent Set (IS). Such a set consists of nodes that are not neighbours of each other. If the IS is dominating (i.e., if all non-IS nodes have a neighbouring IS node) then the IS is said to be maximal. Hence every node in a maximal independent set (MIS) is separated from the nearest MIS node by either two or three hops. MIS-based algorithms are widely used for approximating an MCDS: form an MIS over the network and then connect the MIS nodes together by adding more nodes to the set. Since by definition an MIS is a dominating set, connecting the MIS nodes will form a CDS; the fewer the number of nodes added in the connection phase, the smaller the size of the CDS that is formed.

Alzoubi et al. The algorithm proposed in [2] starts with the construction of a spanning tree which has a root node. The nodes are lexicographically ordered by their tree level (which is defined as the number of levels it lies away from the root node) and by their node identifier. The MIS construction phase begins with the inclusion of the root node. Nodes are excluded from the MIS if a neighbour is included in the MIS; for instance, all neighbours of the root node are excluded. Conversely, a node is included in the MIS if all of its lower-ranked neighbours are excluded. When the MIS construction phase is complete, the root node initiates the joining phase by joining a CDS and sending out a join invitation to the nearest MIS nodes. When an MIS node receives a join invitation, it joins the CDS and resends the invitation to its nearest MIS nodes. Non-MIS nodes also join the CDS when they relay the join invitation to MIS nodes that have not yet joined the CDS. This algorithm has a guaranteed size of at most 8 times the size of the MCDS. The algorithm has a time complexity of $O(n)$ and a message complexity of $O(n \log n)$.

Cardei et al. In contrast to the aforementioned algorithm, the algorithm in [5] immediately starts with the MIS construction phase. A designated initiator node is included in the MIS, and the other nodes in the network are classified for inclusion or exclusion, depending on the classification of their neighbouring nodes. A node is excluded from the MIS if they have a neighbour included in the MIS. A node is a candidate for inclusion in the MIS if they are unclassified and at least one neighbour is excluded. A candidate node becomes part of the MIS if they cover more unclassified neighbours than their candidate neighbours. The joining phase is facilitated by a distributed depth-first traversal, in which a join token is forwarded by nodes in the MANET to ensure that every node in the graph is dominated by a node in the resulting CDS. The strategy in the joining phase is to include non-MIS nodes that have many MIS neighbours. The phase begins with the inclusion of the initiator node in the CDS. A node is considered to be unexplored if it has not directly received the join token. When a node receives a join token, it will forward the join token to one of its neighbours. An MIS node will choose the unexplored non-MIS neighbour that connects to the most MIS nodes without a dominator, while a non-MIS node will pass it to any unexplored MIS neighbour. Neighbours who receive the token but who are not the intended recipient of the token will join the CDS and set its dominator to the sender of the message. If a node belongs to the MIS, a node will send a done message to its dominator when all of MIS nodes that connect to its non-MIS neighbours are dominated. If it is a non-MIS node, a done message is sent when all of its neighbouring MIS nodes have dominators. If a node receives a done message and there is still work to be done, it sends a join token. The process continues until the initiator node has received a done message from all of its neighbours, which indicates that a CDS has been formed. This algorithm has a guaranteed size of at most 8 times the size of the MCDS.

The proposed algorithm is a modification of [5] where the MIS construction in the first phase is modified to reduce the number of overlaps. In this case, a candidate node is included in the MIS if it has a lower ratio of excluded neighbours to unclassified neighbours among its candidate neighbours. If there is a tie, the algorithm selects the candidate neighbour with the greater number of unclassified neighbours.

C. CEC Simulations

We conducted several simulations using ns-2 [16] to compare the different Connected Exact Cover Sets generated by each of the algorithms described above. The different acronyms displayed in the results correspond to:

- MISA MIS-based distributed algorithm in [2],
- MISC MIS-based distributed algorithm in [5],
- MISCE MIS-based distributed algorithm proposed in this paper, which modifies [5],
- GK Centralised algorithm in [12],
- GKE Centralised algorithm proposed in this paper, which modifies [12].

All nodes had a standard transmission range of 250m in a field of $1000 \times 1000 m^2$. To assess the impact of density, the size of the network varied between 20 to 100 nodes and the results displayed are the average values over 50 topologies. Figure 1 shows the different size of CDS obtained by each algorithm, while Figure 2 shows the different Dominating Sums (i.e., the sum of the cardinality of neighbours of each dominator: $\sum_{i \in CDS} |N(i)|$) generated by each corresponding CDS. Finally, Figure 3 shows the overall energy (transmission and reception) costs of a broadcast along the CDS in each case, where each cost corresponds to:

$$ (t_i \times |CDS|) + (r_j \times \sum_{i \in CDS} |N(i)|) $$

using the specifications of the Lucent card studied in [10] (i.e., $t_i = 484mW$ and $r_j = 124mW$).

The results show that our proposed centralised algorithm (GKE) substantially reduces the net-cost of energy compared to the best CDS algorithm (GK) known by reducing the number of overheard receptions (i.e., the Dominating Sums) without increasing dramatically the number of transmissions (i.e., the number of dominators). In fact, it should be noted that the proposed centralised algorithm appears better adapted.
to the unit-disk graph topology compared to [12] as the CDS size is often reduced. This also means that the energy-efficiency result is rarely dependent on the values \( t_i \) and \( r_j \) (i.e., the specifications of the wireless device used) as the Domination Sums are consistently better, and the CDS sizes are rarely worse. Although the energy-cost improvement is less substantial in the distributed algorithm case (MISCE), it demonstrates the need for specific distributed designs that go beyond the optimization of the CDS size.

![Figure 1: Size of Connected Dominating Sets.](image1)

![Figure 2: Domination Sums of CDS.](image2)

IV. CONCLUDING REMARKS

In this paper, we proved that when a realistic model of reception energy costs is considered, the popular CDS scheme for virtual backbone used in broadcast is unrealistic. A variant, proven to be NP-complete in this paper, must be considered, for which we have proposed two algorithms: one centralised which outperforms and one distributed which compares equally with the best known virtual backbone algorithms.

REFERENCES


