ITERATIVE REGION OF INTEREST RECONSTRUCTION IN EMISSION TOMOGRAPHY

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ABSTRACT

This paper proposes a new method for region of interest reconstruction in emission tomography assuming projections from the entire field of view (FOV) are available. Unlike some other region of interest evaluation approaches where the system matrix is reformulated simply by summations over columns corresponding to pixels outside the region of interest, our approach reformulates the system matrix using the Bayes probability formula. The simulation study reveals that our method outperforms the traditional approaches judged by reconstruction quality.

Keywords: Region of interest reconstruction, maximum penalized likelihood, system matrix formulation, Bayes probabilities.

I. INTRODUCTION

This paper considers the problem of region of interest (ROI) reconstruction in emission tomography. Our aim is to develop accurate and easy-to-implement ROI reconstruction algorithms.

In emission tomography, medical interests are often reduced to a particular region or organ, called ROI, and hence it is unnecessary to reconstruct the entire field of view (FOV) covered by the tomographic system. We divide FOV into two regions: the ROI and the region outside the ROI, i.e.

$$\text{FOV} = \text{ROI} \cup \text{region outside ROI}. \tag{1}$$

It is not only computationally more efficient if reconstructing only the ROI, but also it becomes feasible to compute the mean and covariance matrix of the reconstructed ROI due to its much smaller number of voxels.

We must emphasize the important different between our interest in this paper and the focus of region of interest evaluation, for example, in [1] and [2]. Their aim is to conduct ROI evaluation due to concerns in the quantification of emission tomographic data [2]. In this context, the FOV is divided into several ROIs, namely

$$\text{FOV} = \text{ROI}_1 \cup \cdots \cup \text{ROI}_d, \tag{2}$$

and the aim is to estimate the average radiotracer concentration in each ROI.

The following notations are adopted throughout this paper. Assume that FOV contains p voxels and we use $x_1, \ldots, x_p$ to denote radiotracer concentrations of voxels $1, \ldots, p$. Let $y_1, \ldots, y_n$ be camera measurements of camera bins $1, \ldots, n$. We use x to represent the p-vector of all radiotracer concentrations and y to present the n-vector of all camera measurements. The system matrix, with size of $n \times p$, is denoted by A, where its (i,j)th element a_{ij} represents the conditional probability that, given a photon is released from voxel j, it will landed on camera bin i. Let $\mathcal{N}$ be the index set for all voxels, i.e. $\mathcal{N} = \{1, \ldots, p\}$. Also let the index set for the ROI be denoted by $\mathcal{R}$ and the index set for the region outside the ROI be denoted by $\mathcal{O}$, then

$$\mathcal{N} = \mathcal{R} \cup \mathcal{O}. \tag{3}$$

Our interest is to estimate those x_j, where $j \in \mathcal{R}$, from the camera measurements $y_1, \ldots, y_n$.

For simplicity, we assume the tomographic imaging model considered by this paper is the same as [3]. Namely, we assume the y_i's are independent and $y_i \sim \text{Poisson}(\mu_i)$ with $\mu_i = A_i x$, where A_i represents the ith row of A. The positive maximum penalized likelihood (MPL) estimate of x is defined by

$$\hat{x} = \underset{x \geq 0}{\operatorname{argmax}} \left\{ \sum_{i=1}^{n} (-\mu_i + y_i \log \mu_i) - hJ(x) \right\}, \tag{4}$$

where $h > 0$ is the smoothing parameter and $J(x)$ is the penalty function constraining $\hat{x}$ to possess certain local smoothness properties.

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We use $x_{\mathcal{R}}$ to denote the sub-vector of x corresponding to the ROI and $x_{\mathcal{O}}$ the sub-vector corresponding to the region outside ROI. The key factor for accurate estimation of $x_{\mathcal{R}}$ depends largely on accurate formulation of the new system matrix. Methods described in [1] and [2], as briefly explicated in Section II, formulate the system matrix simply by summing the columns of A corresponding to the region outside the ROI. In Section III-A we introduce a new approach to formulate the system matrix, which is based on the Bayes formula. A positively constrained multiplicative iterative algorithm is adopted in Section III-B to provide positive MPL ROI reconstructions. A simulation study is reported in Section IV to demonstrate advantages of our method over the existing methods. Finally, conclusions are given in Section V.

II. SOME EXISTING ROI EVALUATION METHODS

Iterative methods already exist for ROI evalution in emission tomography, such as [1] and [2]. Although these methods can be used for ROI reconstruction, they were developed in a totally different context, as has been explicated in Section I. We summarize some of these method in this section.

According to (2), FOV is divided into d ROIs. Let $\mathcal{R}_1, \ldots, \mathcal{R}_d$ be the index sets for these d ROIs, then $\mathcal{N} = \cup_{r=1}^{d} \mathcal{R}_r$. Note that μ_i can be reexpressed as

$$\mu_i = \sum_{r=1}^{d} \Big(\sum_{j \in \mathcal{R}_r} a_{ij} x_j \Big). \tag{5}$$

If radiotracer concentration in each ROI can be assumed constant, then μ_i in (5) becomes

$$\mu_i = \sum_{r=1}^{d} g_{ir} \xi_r, \tag{6}$$

where ξ_r denotes the common x_j for the rth ROI and $g_{ir} = \sum_{j \in \mathcal{R}_r} a_{ij}$.

Equation (6) explains that the new system matrix $G = (g_{ir})_{n \times d}$ is obtained from A by summing columns of A corresponding to each ROI.

Due to (6), existing emission tomographic reconstruction methods can be applied directly. For example, [1] proposes the maximum likelihood expectation-maximization (ML-EM) [3] to estimate ξ_r, $r = 1, \ldots, d$. [2] argues that as d is small, direct least-squares method, which involves matrix inversion, can be employed to estimate ξ_r.

We may follow the above ROI evaluation methods to perform our ROI reconstruction tasks. Referring to the ROI in equation (1), we assume there are m voxels in the ROI. Then FOV is divided into $m+1$ regions, where the first m regions correspond to m voxels in the ROI and the last region corresponds to the region outside the ROI. Thus the new system matrix G is formulated as:

$$G = (g_{ir})_{n \times (m+1)}, \tag{7}$$

where $g_{ir} = a_{ij}$ for $j \in \mathcal{R}$ and $r \leq m$ (the definition of $\mathcal{R}$ is in (3)), and $g_{i,m+1} = \sum_{j \in \mathcal{O}} a_{ij}$. However, this approach of formulating the new system matrix G (actually its last column) usually leads to low quality ROI reconstructions; see Section IV for an example. Its main reason is that the elements of the last column of G no longer represent true conditional probabilities.

In next section we will explain a new, and correct, approach to compute the last column of the new system matrix.

III. MAXIMUM PENALIZED LIKELIHOOD ROI RECONSTRUCTION

In this section we provide the details of our ROI reconstruction method, including, first, how to formulate the new system matrix correctly, and then how to find the positively constrained MPL reconstruction iteratively.

III-A. System matrix formulation

The key factor for an accurate ROI reconstruction in emission tomography is that the system matrix A must be reformulated to reflect the fact that all voxels outside the ROI are combined to form a single voxel. To differentiate from the system matrix notation used in Section II, we let, in this section, $\tilde{A}$ be the new system matrix for the ROI reconstruction. The size of $\tilde{A}$ is still $n \times (m+1)$, where its last column corresponds to the combined voxels outside the ROI. Elements of $\tilde{A}$ are denoted by $\tilde{a}_{ij}$.

For emission tomography, a_{ij} represents the conditional probability that a photon is detected in camera bin i given it is released from voxel j, and we denote this interpretation by expressing

$$a_{ij} = P(i \,|\, j). \tag{8}$$

Similarly, we use $P_d(i)$ to denote the probability that a photon arrives in camera bin i and $P_v(j)$ to represent the probability that a photon is released from voxel j.

Thus, after combining all voxels in the region outside the ROI into a single voxel (called voxel $(m+1)$), the entries in the last column of $\tilde{A}$ represent $\tilde{a}_{i,m+1} = P(i \,|\, m+1)$. According to the Bayes probability formula,

$$\begin{aligned}\tilde{a}_{i,m+1} &= \sum_{j \in \mathcal{O}} \frac{P_v(j)}{\sum_{t \in \mathcal{O}} P_v(t)} \frac{P(j \,|\, i) P_d(i)}{P_v(j)} \\ &= \sum_{j \in \mathcal{O}} w_j a_{ij}, \end{aligned} \tag{9}$$

where weight w_j is given by $w_j = P_v(j) / \sum_{t \in \mathcal{O}} P_v(t)$, and these weights satisfy $\sum_{j \in \mathcal{O}} w_j = 1$; see equation (3) for the definition of $\mathcal{O}$.

In (9), probabilities $P_v(j)$ (for $j \in \mathcal{O}$), and thus weights w_j, are unavailable and must be estimated. We recommend to estimate w_j by

$$w_j = \frac{x_j^*}{\sum_{t \in \mathcal{O}} x_t^*}, \tag{10}$$

where x_j^* denotes ceratin pre-estimated x_j for $j \in \mathcal{O}$, such as the filtered-backprojection (FBP) estimate or the iterative maximum likelihood (ML) estimate using a small number (such as 10) of iterations.

In summary, we formulate matrix $\tilde{A}$ as follows. The first m columns of $\tilde{A}$ are given directly by those columns of A corresponding to the ROI, and the last column of $\tilde{A}$, according to equation (9), is given by the weighted average of columns of A corresponding to voxels outside the ROI.

If the ROI is small its system matrix $\tilde{A}$ has much less number of columns than A. Hence it becomes feasible to pre-compute and store $\tilde{A}$.

III-B. Multiplicative iterative ROI reconstruction

Let z represent the radiotracer concentration of the combined pixels outside the ROI and let $\theta = (x_{\mathcal{R}}, z)^T$; see Section I for the definition of $x_{\mathcal{R}}$. The positive MPL ROI reconstruction is defined as:

$$\hat{\theta} = \underset{\theta \geq 0}{\operatorname{argmax}}\, \Phi(\theta), \tag{11}$$

where

$$\Phi(\theta) = \sum_{i=1}^{n} (-\tilde{\mu}_i + y_i \log \tilde{\mu}_i) - hJ(\theta), \tag{12}$$

where $\tilde{\mu}_i = \tilde{A}_i \theta$ with $\tilde{A}_i$ being the ith row of $\tilde{A}$, $h > 0$ is the smoothing parameter and $J(\theta)$ is the penalty function. It is important that $J(\theta)$ smoothes only $x_{\mathcal{R}}$ rather than z. For example, for a quadratic penalty $J(\theta) = \frac{1}{2}\theta^T \Gamma \theta$, where the superscript T denotes matrix transpose, we choose

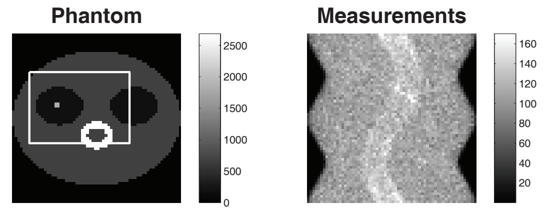

Fig. 1. Phantom (left) and projection measurements (right).

$$\Gamma = \begin{pmatrix} R & 0_{1 \times m} \\ 0_{m \times 1} & 0 \end{pmatrix}. \tag{13}$$

To solve optimization problem (11), the existing positive MPL reconstruction methods, such as [4] and [5], can be adopted directly. For this paper, however, we opt to use the multiplicative iterative (MI) algorithm of [6] to iteratively compute the positive MPL estimate. Let $\theta^{(k)}$ be the estimate of θ at iteration k. At iteration $k+1$, the MI algorithm involves two steps:

1) Compute

$$\theta_j^{(k+1/2)} = \theta_j^{(k)} \frac{\sum_i \tilde{a}_{ij} y_i / \tilde{\mu}_i^{(k)} - hJ_j'(\theta^{(k)})^-}{\sum_i \tilde{a}_{ij} + hJ_j'(\theta^{(k)})^+}, \tag{14}$$

where $\tilde{\mu}_i^{(k)}$ is $\tilde{\mu}_i$ evaluated at $\theta^{(k)}$, $J_j'(\theta)$ represents the derivative of J with respect to θ_j, $a^- = \min(a, 0)$ and $a^+ = \max(a, 0)$.

2) If $\Phi(\theta^{(k+1/2)}) > \Phi(\theta^{(k)})$ then set $\theta^{(k+1)} = \theta^{(k+1/2)}$; otherwise find an $\alpha^{(k)} < 1$ to give

$$\theta^{(k+1)} = \theta^{(k)} + \alpha^{(k)} (\theta^{(k+1/2)} - \theta^{(k)}) \tag{15}$$

such that $\Phi(\theta^{(k+1)}) > \Phi(\theta^{(k)})$. The line search step size $\alpha^{(k)}$ can be obtained by, for example, step half or Armijo's rule.

It has been proved in [6] that, under certain regularity conditions, the MI algorithm converges to the positively constrained MPL solution $\hat{\theta} \geq 0$.

IV. SIMULATION STUDY

We used an elliptical phantom shown in Fig. 1 (left) in this simulation study. The elliptical phantom, which has the dimensions of 64×51 pixels, is contained inside a square of 64×64 pixels. The background outside the elliptical phantom has zero emissions. The two low

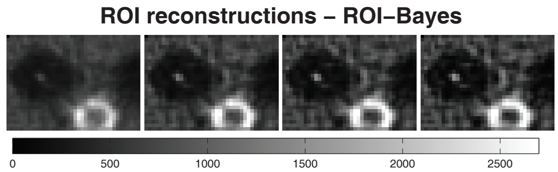

Fig. 2. ROI reconstructions by ROI-Bayes (row 1) and ROI-sum (row 2). Columns correspond to iterations 8, 32, 64 and 128.

activity circles represent the lungs and the high activity ring corresponds to the myocardium. Within the left lung there is a lesion of size 2×2 pixels. The area inside the white rectangle is the ROI.

The simulated system used SPECT geometry. There were 64 attenuated parallel beam projections uniformly spaced over $360°$, and each projection contained 64 measurements. Attenuation coefficients were 0.15 /pixel (water) within the body, except for within the two lungs, where the coefficients were 0.0375 /pixel (vapour). The projection measurements (Poisson noise contaminated) are displayed in Fig 1 (right) and the total measurement (counts) is 401,674.

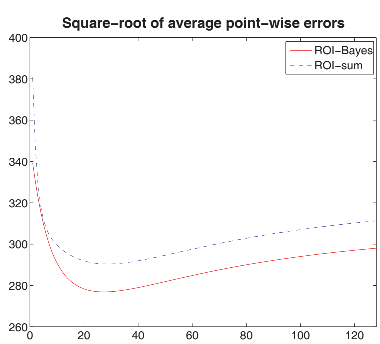

Fig. 3. Plot of square root of average squared errors for ROI-Bayes (-) and ROI-sum (- -).

We performed ROI reconstructions using both the summation (called ROI-sum) and the Bayes (called ROI-Bayes) based system matrix formulations. For fair comparisons, both approaches used the same starting value, the same quadratic penalty matrix (13) and the same smoothing value $h = 2 \times 10^{-6}$. For ROI-Bayes, ten ML-EM iterations were run to estimate the last column of $\tilde{A}$ (see equation (10)). Reconstructions are exhibited in Fig. 2. Clearly, ROI-Bayes produced more accurate estimate judged by the image scale. Plots of square-root of average point-wise squared errors (defined as $(\frac{1}{m}\sum_{j=1}^{m}(\hat{\theta}_j - \theta_{\text{true},j})^2)^{1/2}$) against iteration numbers in Fig. 3 also confirm this finding.

V. CONCLUSION

This paper develops an accurate ROI reconstruction method. The fundamental difference between our and the others is that we use the Bayes formula to compute the new system matrix. The simulation study indicates that our approach produces less reconstruction errors. However, our method requires a brief full FOV reconstruction using, e.g. FBP or ML with a small number of iterations, and hence is slightly more computational demanding.

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