Performance Bounds for Detect and Avoid Signal Sensing

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Abstract—Detect and Avoid (DAA) is a Cognitive Radio technique for a primary user and secondary users sharing the same spectrum. The DAA spectral sensing system must provide both low decision delay and highly reliable decisions on the presence/absence of primary user transmission. These results provide upper and lower bounds on the decision error rate performance of sensing systems as a function of SNR (signal-to-noise ratio) and the number of samples used in the sensing decision. The bounds represent benchmarks on the performances of practical detection systems and give insight into the trade-off between sensing performance and decision delay.

I. INTRODUCTION

Cognitive Radio may provide spectrum sharing capabilities among a primary user, who has priority in the use of the spectrum, and secondary users. In this context, a simple coexistence mechanism is provided by the so-called Detect and Avoid (DAA) operation [1]. Basically, a secondary user may use the spectral band during times that it is unoccupied by the primary user. However, the secondary user must quickly and accurately sense the presence of the primary user and avoid transmission when the primary user is transmitting.

A fundamental issue arises regarding how much primary user signal structure should be used in DAA signal processing. The simplest form of detect and avoid signal processing uses energy sensing. The most complex form of detect and avoid signal processing uses waveform sensing, which utilizes the complete detail of the primary user waveform, including modulated data. The decision error performance of any practical detect and avoid signal sensing is upper bounded by the energy detector error performance and lower bounded by the waveform detector error performance. Therefore these bounds expressed as a function of signal to noise ratio and the number of time samples used in a primary user detection decision of are great practical interest in the design of cognitive radio systems. The difference between the upper and lower bounds is also important in the determination of how much primary user signal structure to use in DAA processing.

It is assumed that the complex envelope of the channel output is time sampled at a rate of $R$ complex samples per second and that the samples are statistically independent. $N_g = T_g B$ samples are used for the primary user detection, where, $T_g$ is the decision delay time in seconds and $B$ is the primary user signal bandpass bandwidth in Hertz.

Using the Sampling Theorem, $R = B$ samples per second. Therefore, for a primary user signal with bandwidth $B$, $N_g$ is directly proportional to the signal observation time.

For successful DAA operation, it is critically important that the presence and absence of the primary user signal is quickly and accurately detected by the secondary user receivers. Sensing delay must be traded off against sensing accuracy in the system design.

The relationship between the number of time samples and signal-to-noise ratio required to achieve a particular level of sensing performance was presented in Reisenfeld and Maggio [2]. These results apply to any modulation type, including constant envelope modulation and Gaussian signal modulation. Gaussian signal modulation, in which time samples of the modulated signal are Gaussian random variables, models OFDM. There are many cases of practical interest in Cognitive Radio in which the primary user modulation is OFDM. The results in Reisenfeld and Maggio[2] are summarized in this paper. This paper also provides new simulation results relating to the maximum likelihood signal sensing error probability for Gaussian modulation, such as OFDM. The results are compared to the analytically derived error probability expressions for both energy sensing and waveform sensing.

II. SPECTRAL SENSING MODEL

The model described by Tang [3] for spectral sensing was to process either a time domain sequence of channel output complex envelope time samples or a frequency domain set of FFT coefficients. The detection of the primary spectrum user signal can be characterized by the binary hypothesis test, where $H_0$ is the hypothesis representing the primary signal not being present and $H_1$ is the hypothesis representing the primary signal being present. A zero mean, additive white Gaussian noise channel is assumed. The receiver processes the channel output to obtain discrete samples of the complex envelope or, alternatively, FFT output complex coefficients. The number of complex channel output samples that are processed for a detection decision is $N_g$. Therefore,

$$
H_0: \quad y(n) = z(n), \quad n = 1, 2, \ldots, N_g
$$

$$
H_1: \quad y(n) = x(n) + z(n), \quad n = 1, 2, \ldots, N_g
$$

(1)

where, $y(n)$ are time samples of the complex envelope of the channel output, $x(n)$ are time samples of complex envelope of the primary user signal, and $z(n)$ is a sequence of statistically independent, zero mean, complex Gaussian random variables with variance equal to $\sigma_z^2$. 
Alternatively, \( y(n) \) could represent the FFT output coefficients resulting from an input of a sequence of time samples of the complex envelope of the channel output.

### A. Energy Sensing

The energy metric, \( S \), is defined as,

\[
S = \sum_{n=1}^{N_s} |y(n)|^2
\]

Define, \( \mu_{S} = E[S \mid H_0] \), \( \mu_{S} = E[S \mid H_1] \), \( \sigma^2_{S} = \text{Var}[S \mid H_0] \), \( \sigma^2_{S} = \text{Var}[S \mid H_1] \),

and,

\[
\alpha = \frac{\sqrt{\text{SNR}}}{2}\left(\frac{\sigma^2_0}{\sigma^2_0 + \sigma^2_1}\right)
\]

Also define the decision threshold, \( S_T \), such that the binary decision rule for spectrum sensing is:

If \( S \geq S_T \), then decide \( H_1 \),
If \( S < S_T \), then decide \( H_0 \).

Then two performance measures that characterize the detection performance are,

\[
P_{FD} = \text{Pr}\{\text{false detection}\} = \text{Pr}\{S \geq S_T \mid H_0\}
\]

and,

\[
P_{MD} = \text{Pr}\{\text{missed detection}\} = \text{Pr}\{S < S_T \mid H_1\}
\]

If \( N_s \) is sufficiently large, \( S \) is essentially Gaussian because of the Central Limit Theorem. Under the Gaussian assumption,

\[
P_{FD} = Q\left(\frac{S_T - \mu_{S_0}}{\sigma_{S_0}}\right)
\]

and,

\[
P_{MD} = Q\left(\frac{\mu_{S_1} - S_T}{\sigma_{S_1}}\right)
\]

where,

\[
Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz
\]

For maximum likelihood detection,

\[
P_{FD} = P_{MD}
\]

and, the detection threshold for maximum likelihood detection is,

\[
S_T = \frac{\mu_{S_0}\sigma_{S_1} + \mu_{S_1}\sigma_{S_0}}{\sigma_{S_0} + \sigma_{S_1}}
\]

Using the maximum likelihood decision threshold, the **sensing error floor (SEF)** is defined [3] as,

\[
SEF = P_{FD} = P_{MD} = Q\left(\frac{\mu_{S_0} - \mu_{S_1}}{\sigma_{S_0} + \sigma_{S_1}}\right)
\]

The signal sequence, \( \{x(n)\} \), may be modeled as an ergodic random process. As shown in Reisenfeld and Maggio[2], for the energy detection statistic defined in (2), the evaluation of (3), (4), (5), and (6) in (14) results in,

\[
SEF = Q\left(\frac{\sqrt{N_b} \sqrt{\text{SNR}}}{1 + \sqrt{(\alpha - 1) \text{SNR}^2 + 2 \text{SNR} + 1}}\right)
\]

where,

\[
\alpha = \frac{\sqrt{\text{SNR}}}{2}\left(\frac{\sigma^2_0}{\sigma^2_0 + \sigma^2_1}\right)
\]

and, the signal-to-noise ratio, \( \text{SNR} \), is defined as,

\[
\text{SNR} = \frac{\mathbb{E}[|x(n)|^2]}{\sigma^2_2}
\]

\( \alpha \) is a parameter that characterizes the shape of the probability density function of the magnitude of the complex envelope of the modulation. For constant envelope modulation, \( \alpha = 1 \). If \( |x(n)| \) is Rayleigh distributed which is the case of Gaussian modulation, \( \alpha = 2 \). For OFDM modulation, \( |x(n)| \) is asymptotically Rayleigh distributed as the number of sub-carriers becomes large.

Evaluating (9) and (10) for the energy detection results in,

\[
P_{FD} = Q\left(\frac{S_T}{\sqrt{N_b} \sqrt{\sigma^2_2}} - \sqrt{N_b}\right)
\]

and,

\[
P_{MD} = Q\left[\frac{\sqrt{N_b} (\text{SNR}+1) - S_T}{\sqrt{(\alpha - 1) \text{SNR}^2 + 2 \text{SNR} + 1}\sqrt{\sigma^2_2}}\right]
\]

\( P_{FD} \) and \( P_{MD} \) may be traded off for fixed \( \text{SNR} \) and \( N_b \) by varying the decision threshold, \( S_T \).

### B. Waveform Sensing

Waveform sensing uses the complete functional description of the signal to be detected, including the data in the modulated signal. For waveform detection, the detection statistic, \( S \), is given by,

\[
S = \text{Re} \left[ \sum_{n=1}^{N_s} y(n)x^*(n) \right]
\]

where, \( x^*(n) \) is the complex conjugate of \( x(n) \).

In agreement with Tang [3] for the waveform sensing statistic given in (20), the evaluation (3), (4), (5), and (6) in (14) results in the exact equation for \( \text{SEF} \) given by,

\[
SEF = Q\left(\frac{\sqrt{\text{SNR}}}{\sqrt{(\alpha - 1) \text{SNR} + 1/2} + 1/2}\right)
\]

Also, evaluating (9) and (10) for waveform detection results in,
Equations (15), (18), (19), (21), (22) and (23) use the maximum likelihood threshold, $T_S$, given by (13) for the ensemble of modulated waveforms. This threshold is fixed by $\mu_\alpha, \mu, \sigma_x, \sigma_\alpha$. 

### III. EXACT MISSED DETECTION AND FALSE DETECTION PERFORMANCE FOR ENERGY SENSING AND $\alpha = 1$

The detection error performance for energy sensing in Section I used the assumption that $N_\alpha$ was sufficiently large such that detection statistic was essentially Gaussian distributed. A comparison of (15) to the exact energy sensing $SEF$ for small $N_\alpha$ is therefore important. 

Under $H_a$, $S$ has a central chi squared distribution with $2N_\alpha$ degrees of freedom [4], and under $H_1$ with the assumption of constant envelope modulation, $\alpha = 1$ and $S$ has a non-central chi squared distribution with $2N_\alpha$ degrees of freedom [5]. Using these distributions, the exact performance results for energy detection may be obtained as,

$$P_{SD} = Q \left( \frac{S_\alpha}{\sqrt{N_\alpha} \sigma_\alpha^2 \sqrt{SNR}} \right)$$

and,

$$P_{MD} = Q \left[ \frac{\sqrt{N_\alpha} \sigma_x^2 SNR - S_\alpha}{\sqrt{\alpha - 1} SNR^2 + \frac{1}{2} SNR} \right]$$

Equations (15), (18), (19), (21), (22) and (23) use the maximum likelihood threshold, $S_\alpha$, given by (13) for the ensemble of modulated waveforms. This threshold is fixed by $\mu_\alpha, \mu, \sigma_x, \sigma_\alpha$. 

The normalized threshold for the $SEF$ may be obtained by numerically solving (27) for $\frac{S_\alpha}{\sigma_\alpha^2}$. 

This maximum likelihood threshold, which does not utilize a Gaussian approximation, can then be used in to obtain the $SEF$ performance. The threshold can either be used analytically in (24) and (25) or can be used in simulation.

It was verified that for $\alpha = 1$ and $N_\alpha = 10$, there was close agreement between the $SEF$ obtained using the Gaussian approximation and the $SEF$ obtained using the actual distributions of the decision statistic.

$$P_{SD} = \Pr \{ S < S_\alpha \mid H_1 \} = 1 - Q_{\nu_\alpha} \left( \frac{\sqrt{2N_\alpha} \sigma_\alpha^2}{\sqrt{S_\alpha}} \right)$$

where, $Q_{\nu_\alpha}(a, b)$ is the generalized Marcum’s Q function defined by,

$$Q_{\nu_\alpha}(a, b) = \int_{x = 0}^{\infty} \left( \frac{a}{b} \right)^{\nu_\alpha - 1} e^{-\frac{x^2 + a^2}{2}} I_{\nu_\alpha - 1}(ax) dx$$

and, $I_{\nu_\alpha}(x)$ is the $\nu_\alpha$-order modified Bessel Function of the first kind.

### III. THE SENSING ERROR FLOOR ($SEF$) PERFORMANCE FOR ENERGY SENSING FOR $\alpha = 1$

For $\alpha = 1$, it is possible to numerically find the maximum likelihood decision threshold without using the Gaussian approximation for the decision statistic.

Since $SEF = P_{SD} = P_{SD}$, equations (24) and (25) can be used to obtain the decision threshold required for the $SEF$. Therefore,

$$1 - Q_{\nu_\alpha} \left( \frac{\sqrt{2N_\alpha} \sigma_\alpha^2}{\sqrt{S_\alpha}} \right) = e^{-\frac{S_\alpha}{\sigma_\alpha^2}} \sum_{k = 0}^{\infty} \frac{\left( \frac{S_\alpha}{\sigma_\alpha^2} \right)^k}{k!}$$

The normalized threshold for the $SEF$ may be obtained by numerically solving (27) for $\frac{S_\alpha}{\sigma_\alpha^2}$. 

This maximum likelihood threshold, which does not utilize a Gaussian approximation, can then be used in to obtain the $SEF$ performance. The threshold can either be used analytically in (24) and (25) or can be used in simulation.
Fig. 1. The SEF performance for energy sensing and waveform sensing as a function of SNR in dB for $N_a = 100$. The solid lines are simulated results and the dashed lines are analytic results.

IV. NUMERICAL EXAMPLES

In this Section, specific numerical examples of the results of Sections I-III are given. The SEF was obtained as a function of SNR for $\alpha = 1$, which is constant envelope modulation, and $\alpha = 2$, which is Gaussian modulation, by both analysis and computer simulation. Results are given for both energy sensing and waveform sensing.

Figures 1 and 2 show the results for $N_a = 100$ and $N_a = 1000$, respectively, for the energy sensing and waveform sensing cases. For energy sensing, with $N_a = 100$, there is reasonably close agreement between the analysis and simulation for $SEF \geq 10^{-5}$. For the $N_a = 1000$ energy sensing case and for all values of $N_a$ for waveform sensing there is excellent agreement between analysis and simulation.

Figures 3 and 4 show the theoretically required $N_a$ as a function of SNR in dB to achieve $SEF$ equal to $10^{-5}$ and $10^{-6}$, respectively, for $\alpha = 1$.

Figure 5 shows the theoretical SEF performance as a function of SNR in dB, for $N_a = 10$ and $\alpha = 1$. Both the Gaussian approximation and the exact probability density function cases are shown.

Figures 6 and 7 show the SEF performance as a function of SNR in dB, for $N_a = 10$ and $\alpha = 2$, which is the Gaussian signal case, for energy sensing and waveform sensing, respectively. For energy sensing in Figure 6, there is good
Fig. 5. The SEF as a function of SNR in dB for the energy detection with $N_g=10$ and $\alpha=1$. Curves for both the cases of the Gaussian approximation and exact results are shown.

Fig. 6. The SEF performance for energy sensing as a function of SNR in dB for $N_g=10$ and $\alpha=2$. The solid line is an analytic result and the dashed line is a simulated result.

Fig. 7. The SEF performance for waveform sensing as a function of SNR in dB for $N_g=10$ and $\alpha=2$. The solid line is an analytic result and the dashed line is a simulated result.

Fig. 8. The SEF performance for energy sensing as a function of SNR in dB for $N_g=100$ and $\alpha=2$. The solid line is an analytic result and the dashed line is a simulated result.

Fig. 9. The SEF performance for waveform sensing as a function of SNR in dB for $N_g=100$ and $\alpha=2$. The solid line is an analytic result and the dashed line is a simulated result.

agreement between theory and simulation for SEF $\geq 5 \times 10^{-3}$. For high SNR, the large difference between theory and simulation is due to the detection statistic not being Gaussian distributed and the decision threshold not being optimized.

The asymptotic convergence of the SEF to a non-zero value for increasing SNR is verified in Figures 6 and 7.

For Figure 7, which is waveform sensing, there is reasonable agreement between theory and simulation.

Figures 8 and 9 show the SEF performance as a function of SNR in dB, for $N_g=100$ and $\alpha=2$, which is the Gaussian signal case, for energy sensing and waveform sensing, respectively. For larger values of $N_g$, the detection statistic tends towards a Gaussian distribution because of the Central Limit Theorem. There is good agreement between theory and simulation.

V. CONCLUSION

This paper has presented results on the relationship between the detection performance and decision delay time for both energy sensing and waveform sensing. The tradeoff is parameterized on $N_g$, the number of time samples per decision.

Increasing $N_g$ both increases the DAA decision delay time and decreases the SEF. Therefore, $N_g$ needs to be selected in a systematic tradeoff for which the Cognitive Radio interference mitigation is optimized.

The performances of all DAA systems are bounded by the energy and waveform sensing performances. The SEF
results provide benchmarks for practical spectral sensing systems.

![Fig. 9. The SEF performance for waveform sensing as a function of SNR in dB for N_2 =100 and α=2. The solid line is an analytic result and the dashed line is a simulated result.](image)

The results are general and apply to any modulation type. The cases, \( \alpha = 1 \), for constant envelope modulation, and \( \alpha = 2 \), for Gaussian modulation, such as OFDM, are of special interest. For these cases, the analytical and simulation results were compared. There is close agreement in the SEF performance obtained from analysis and simulation.

REFERENCES


