Nonlinear pulse propagation at zero dispersion wavelength in anti-resonant photonic crystal fibers

A. Fuerbach, P. Steinvurzel, J. A. Bolger, and B. J. Eggleton

Centre for Ultrahigh-bandwidth Devices for Optical Systems (CUDOS)
School of Physics, University of Sydney, NSW 2006, Australia
egg@physics.usyd.edu.au

Abstract: We experimentally and numerically investigate femtosecond pulse propagation in a microstructured optical fiber consisting of a silica core surrounded by air holes which are filled with a high index fluid. Such fibers have discrete transmission bands which exhibit strong dispersion arising from the scattering resonances of the high index cylinders. We focus on nonlinear propagation near the zero dispersion point of one of these transmission bands. As expected from theory, we observe propagation of a red-shifted soliton which radiates dispersive waves. Using frequency resolved optical gating, we measure the pulse evolution in the time and frequency domains as a function of both fiber length and input power. Experimental data are compared with numerical simulations.

©2005 Optical Society of America

OCIS codes: (060.5530) Pulse propagation and solitons; (060.7140) Ultrafast processes in fibers; (230.3990) Microstructure devices

References and links
1. Introduction

Photonic crystal fibers (PCFs), optical fibers with a periodic array of air holes in the cladding, have been used for a wide variety of nonlinear optical experiments [1-8]. Solid core PCFs guide by total internal reflection at the interface between the silica core and the low effective index holey cladding [9]. They offer a distinct advantage over conventional step index fibers made from doped glass in that one may more easily achieve a high index contrast between the index holey cladding [9]. They offer a distinct advantage over conventional step index fibers immersed in different liquids, [10-12], though all-solid fibers have also been recently demonstrated schematically in Fig. 1(a), can easily be created by filling the air holes of a conventional PCF with a high index fluid [10-12], though all-solid fibers have also been recently demonstrated [13, 14]. These fibers cannot guide by total internal reflection because of the inverted index contrast. However, the periodic cladding structure does form a photonic bandgap material, and so frequencies of light which fall within one of the bandgaps are forbidden from...
propagating in the cladding and are therefore confined to the core; frequencies which fall outside the gaps are not confined and radiate through the cladding.

It has been shown that the width and center frequency of the bandgaps are dominated by the resonant properties of an individual cylinder and not the geometry of the microstructure as a whole [15], and that mode confinement is achieved through anti-resonant backscattering [16]. To first order, the center frequency of the transmission bands is solely determined by the refractive index contrast and diameter of the cladding cylinders. These high index inclusion PCFs are in fact a cylindrical analog of the planar anti-resonant reflecting optical waveguide (ARROW) [17], and so we generically refer to them as ARROW-PCFs.

Fig. 1. (a) Schematic of ARROW-PCF geometry (b) measured (black) and simulated (red) transmission through ARROW-PCF sample and corresponding simulated $\beta_2$, and (c) electron microscope image of PCF used in experiment.

The resonant nature of the waveguiding mechanism in ARROW-PCFs gives rise to strong dispersive properties. Figure 1(b) plots the group velocity dispersion, $\beta_2$, in three transmission bands of an ARROW-PCF. The dispersion in each band has a similar shape, approaching $+\infty$ at the high frequency edge of the band and $-\infty$ at the low frequency edge, having a positive dispersion slope $\beta_3$ across the band, and an inflection point near the zero dispersion wavelength $\lambda_{ZD}$ [10, 13]. One can thus engineer $\beta_2$ at a given wavelength by filling the air holes with a material having the appropriate refractive index. Similar ideas using index-guided fiber tapers immersed in heavy water [18] or various organic fluids [19] to change the waveguide dispersion have recently been demonstrated. Furthermore, for PCFs with a moderate air fill fraction ($<50\%$), the effective area ($A_{eff}$) of the fundamental mode of an ARROW-PCF can be as small or smaller than $A_{eff}$ in the equivalent air/silica PCFs [20].

In this paper, we provide for the first time a detailed analysis of nonlinear femtosecond pulse propagation near $\lambda_{ZD}$ of one of the transmission bands of an ARROW-PCF. We begin with a brief review of its linear properties and compare multipole method simulations [21] with experimental transmission and dispersion measurements. We then present experimental frequency-resolved optical gating (FROG) measurements of femtosecond pulse propagation through the ARROW-PCF, where we vary the fiber length and pulse peak power, but fix the pulse center wavelength near $\lambda_{ZD}$. For small $\beta_2$, nonlinear propagation is dominated by $\beta_3$, and we observe a soliton shifted in frequency towards the anomalous dispersion regime (red shift for positive $\beta_3$) which radiates dispersive waves in the normal dispersion regime [22, 23]. We compare the obtained FROG traces with numerical simulations of the nonlinear Schrödinger equation (NLSE) [24].
2. Linear characterization

An electron microscope image of the fiber used in the experiments is shown in Fig. 1(c). The fiber (Blaze Photonics SC-5.0-1040) has an average hole diameter of 1.70 µm, a pitch of 3.2 µm, and a core diameter of 4.7 µm formed by a single missing cylinder. Using a vacuum pump, we filled a 35 cm length of this fiber with a commercial index fluid from Cargille Laboratories having \( n_D = 1.62 \). This index fluid was specifically chosen to provide a transmission band centered around 800 nm, covering the tuning range of the Ti:Sapphire laser used in the nonlinear experiments described in Sec. 3.

The linear properties of the fiber were simulated by solving for the complex-valued effective index of the guided modes as a function of wavelength using the multipole method [21]. Figure 1(b) shows the measured transmission through the sample (black lines) and the transmission predicted by the multipole simulations (red lines). The experimental spectra are obtained using a supercontinuum white light source [25] based on the same fiber we use to fabricate the ARROW-PCF and an optical spectrum analyzer (OSA). They are normalized against transmission through the same fiber with no fluid. The simulated spectra are derived from the imaginary part of the effective index. We note that unlike the simulated transmission bands, the measured transmission bands support several resonant loss dips. While the physical origin of these dips is not yet entirely understood, they appear (shifted in frequency) in the measured spectra of fibers filled with different fluids, indicating that they are a feature of the ARROW-PCF geometry and not absorption resonances of the materials.

We measure the dispersion of our ARROW-PCF in the band centered around 800 nm using a Mach-Zehnder interferometer [10, 13], shown schematically in Fig. 2(a). The frequency spacing between adjacent peaks in the measured interference pattern is inversely proportional to the group delay (\( \tau_D \)) of a pulse propagating through the ARROW-PCF sample [10]. Figure 2(b) shows a plot of \( \tau_D \) and \( \beta_2 \) vs. wavelength obtained from measurement and simulations. The black squares and solid black line correspond to the measured delay and a 4th order polynomial fit to the measurement, while the blue line corresponds to \( \beta_2 \) derived from the measurement of \( \tau_D \); the red line shows \( \beta_2 \) derived from the multipole simulations. We note...
that the $\lambda_{ZD}$ shown here at 773 nm differs from the value reported in [20] of 762 nm, even though we have used the same index fluid and fiber from the same spool. We attribute this to variations in the average hole size along the fiber length (1.70 $\mu$m in the present case versus 1.67 $\mu$m in [20]). The revised hole size is in fact derived from the shift in $\lambda_{ZD}$ and the corresponding shift in the transmission bands. We have confirmed that the discrepancy is not due to contamination or degradation of the index fluid, and in fact have found our samples to be quite stable over several months even though no effort was made to seal the fiber and protect the fluid from the outside environment. Samples made with fiber from further along the spool show a further red shift of $\lambda_{ZD}$ and a corresponding shift in the transmission bands. Such variations in hole size are within the specifications of this fiber, which is designed for supercontinuum generation using nanosecond pulses (without the fluid); a change in hole size from 1.67 to 1.70 $\mu$m in the air hole fiber blue shifts $\lambda_{ZD}$ by only 2.5 nm and very slightly shifts the dispersion slope upward. In the present case (filled with fluid), each hole acts as an optical resonator, and we also probe the structure with extremely short pulses, so we are far more sensitive to variations in the structural dimensions.

3. Nonlinear pulse propagation

When an ultrashort pulse propagates nonlinearly near $\lambda_{ZD}$ in an optical fiber, the input spectrum separates into two well-separated components [22, 23]. When $\beta_2$ is small, the pulse experiences strong self phase modulation (SPM) which broadens its spectrum. Higher order dispersion terms are non-trivial in this case, so as the pulse bandwidth broadens, different parts of the spectrum experience different $\beta_2$; in the case where $\beta_3$ is the dominant term, opposite sides of the pulse spectrum experience normal or anomalous $\beta_2$. On the anomalous side, the pulse spectrum continues to broaden until $\beta_2$ becomes large enough to roughly balance SPM, at which point we obtain a stable soliton with a pronounced shift in center frequency. However, the soliton is nonlinearly phase-matched to a dispersive wave in the normal dispersion regime. As the pulse propagates along the fiber and experiences its initial frequency shift, it continuously radiates dispersive waves until the stability point is reached. This process has been shown to be analogous to Cherenkov radiation [26] and plays a key role in supercontinuum generation in optical fibers [27].

In our experiment, we characterize the temporal and spectral evolution of a femtosecond pulse as a function of both fiber length (cutback) and pulse peak power. A schematic of the experimental setup is shown in Fig. 3. We use a passively mode-locked Ti:Sapphire laser (Spectra Physics Tsunami) which provides 70 fs pulses at 80 MHz with the center wavelength $\lambda_0$ tuned to 775 nm, near $\lambda_{ZD}$ of the ARROW-PCF. The pulses go through a Faraday isolator and a prism pair, which allows compensation of any material dispersion present in the beam path. Two half-wave plates and a broadband polarizer are used to adjust the power and (linear) polarization state of the laser pulses and a Kepler-type 1:2 telescope expands the beam. A 25X microscope objective focuses the beam into the ARROW-PCF, which is mounted on an xyz-translation stage. At the fiber output, an achromatic lens collects the transmitted light pulses which are sent to a spectrometer (Ocean Optics HR2000) and a FROG device [28]. The latter utilizes a 200 $\mu$m thick BBO crystal for second harmonic generation and the same spectrometer for recording the FROG spectrogram of the upconverted pulse.
For the cutback measurements, we fix the average input power at 30 mW and cut the fiber back from 350 mm down to (almost) zero in 20-50 mm steps. Figure 4(a) shows the evolution of the FROG trace for increasing fiber length; the first frame corresponds to the output directly from the laser. In ARROW-PCFs, $\beta_3$ is always positive, so we expect the soliton-like component of the spectrum to be red-shifted from the input pulse and the dispersive wave to be blue-shifted. In the FROG trace, we see the optical intensity is initially well confined both in time (horizontal axis) and frequency (vertical axis). As the fiber length increases, the intense part of the FROG trace shifts down in frequency but remains well confined in time, corresponding to the soliton-like component which we expect to move towards the anomalous dispersion regime. At the same time, we see blue shifted radiation grow out of the soliton. This part of the spectrum is stretched along the time axis with increasing length, indicative of a dispersive wave.

In the second experiment we fix the fiber length to 180 mm and vary the average input power between zero and 70 mW. For higher input powers, we observed damage of the fiber input. We attribute this to heating of the fiber, which causes the fluid to expand and leak out over the end face, where it nonlinearly absorbs the input pulse and burns, damaging the fiber core. However, we note that our multipole simulations indicate that 97.5% of the fundamental mode energy lies within the silica core, and so we assume that the nonlinear properties of the fluid have no bearing on our pulse propagation measurements. The power measured at the output of the fiber increases linearly with the input power, indicating that there is no multiple photon absorption. Also, any possible fluid nonlinearity is not included in the NLSE simulations described below, and the results agree quite well with experiment. Figure 4(b) shows the evolution of the FROG trace for increasing power. The results are similar to the cutback results, and we observe a soliton and growth of a dispersive wave with increasing power. One notable difference is that the center frequency of the soliton shifts monotonically towards longer wavelengths, with no indications of saturation. At higher powers, in fact, we expect the shift to continue to increase due to the combined effects of $\beta_3$ and intrapulse Raman self-frequency shift. We return to this point in Section 4.
Fig. 4. Movie of the evolution of the FROG Trace with (a) increasing fiber length (0 – 350 mm) and fixed average power (30mW) (370 kB) and (b) fixed fiber length (180 mm) and increasing average power (0 – 70 mW) (334 kB).

We simulated pulse propagation in the fiber by numerically solving the NLSE using the split-step Fourier method [24]. Our model accounts for dispersive terms up to the fourth order, the delayed Raman response, and the effect of self-steepening. Due to the relatively short fiber length, propagation losses are neglected. The parameters we use for the ARROW-PCF are \( \beta_2 = -5.62 \text{ ps}^2/\text{km}, \beta_3 = 0.65 \text{ ps}^3/\text{km}, \beta_4 = -5.54 \times 10^{-4} \text{ ps}^4/\text{km}, \) and the nonlinear parameter \( \gamma = 0.0162 (\text{W} \cdot \text{m}^{-1}). \) Our simulations show that the pulse evolution is dominated by \( \beta_3 \) and independent

Fig. 5. (a) Spectral and (b) temporal evolution of the pulses as they propagate inside the ARROW-PCF. The average input power is fixed at 30 mW. Left pictures: Results obtained from NLSE simulations. Right pictures: Data retrieved from the measured FROG-Traces.
Fig. 6. (a) Spectral and (b) temporal intensity and phase of the ultrashort laser pulses after propagation in a 250 mm long ARROW-PCF. Red line: Direct measurement of the spectral intensity. Blue lines: Retrieved from the measured FROG-Trace. Black lines: Results of the NLSE simulations.

Fig. 7. (a) Spectral and (b) temporal evolution of the pulses in the ARROW-PCF as a function of input power. The fiber length is fixed at 180 mm. Left pictures: Results obtained from NLSE simulations. Right pictures: Data retrieved from the measured FROG traces.
of $\beta_2$ and $\beta_4$ provided these are small ($0 < \beta_2 < 10 \text{ ps}^2/\text{km}, |\beta_4| < 5 \times 10^{-3} \text{ ps}^4/\text{km}$). The values for the dispersive terms were initially taken from the dispersion measurement shown in Fig. 2(b), with $\beta_3 = 0.76 \text{ ps}^3/\text{km}$. Again, the pulse evolution is extremely sensitive to the value of $\beta_3$, and we found that the simulation best approximates the measurement for $\beta_3 = 0.65 \text{ ps}^3/\text{km}$, which is well within our measurement error of $\pm 0.20 \text{ ps}^3/\text{km}$ (we may in fact consider the femtosecond pulse propagation experiments as a more sensitive measurement of $\beta_3$). We note that multipole simulation in Fig. 2(b) gives a somewhat larger $\beta_3$ near 775 nm ($1.08 \text{ ps}^3/\text{km}$) and that this value yields NLSE simulation results which widely differ from the measurement. We attribute this discrepancy to small variations in the inter-hole spacing and hole size in the experimental fiber microstructure, whereas the multipole simulation assumes uniform hole size and spacing. While such structural asymmetries may have only a minor effect on the transmission spectra of ARROW-PCFs, it is quite likely that they greatly affect the higher order dispersive terms (indeed, we find a 1.7% increase in average hole size shifts $\lambda_{ZD}$ by 11 nm). More detailed simulations that quantify these effects will be the subject of future work.

Figures 5 and 7 compare the temporal and spectral evolution of the pulse propagation in both experiments as obtained from the simulations (left side) and extracted from the FROG traces (right side). Figure 6 plots the slices at $z=250$ mm from Fig. 5, along with the phase of the pulse and a direct measurement of the spectrum. The very steep trailing edge of the pulse results from self-steepening. As expected, the phase is relatively flat across the soliton-like component of the spectrum and strongly varying across the dispersive component. Finally, we note that the directly measured spectrum agrees well with the spectrum extracted from the FROG trace and the simulation.

4. Discussion

When characterizing nonlinear propagation effects in fibers, it is useful to consider the two canonical length scales, the nonlinear length $L_{NL}$ and dispersion length $L_D$ defined as

$$L_{NL} = \frac{1}{\gamma \cdot \hat{P}} \quad \text{and} \quad L_D = \frac{\tau_0^2}{|\beta_2|}$$

(1)

where $\hat{P}$ is the pulse peak power, $\tau_0 = \tau_p / 1.7603$, and $\tau_p$ is the FWHM pulse duration of the laser pulse which is assumed to be sech$^2$-shaped. When we consider the case of soliton propagation in the anomalous dispersion regime, the soliton order is given by

$$N = \left[ \frac{L_D}{L_{NL}} \right]^{1/3}$$

(2)

For our fiber, $\beta_3$ is very small, hence the pulse propagation is governed by $\beta_3$. It is therefore useful to define the additional parameters [22, 24]

$$L_D' = \frac{\tau_0^3}{|\beta_3|} \quad \text{and} \quad N' = \left[ \frac{L_D'}{L_{NL}} \right]^{1/3}$$

(3)

Our numerical simulations show that for the power range we investigated, the $\beta_3$-induced frequency shift $\Delta \omega$ of the soliton-like component of the spectrum can be estimated by demanding the steady state to be a first order soliton with $N=1.5$, using $L_D$ at the shifted frequency with $\beta_3 (\omega) = \beta_3 (\omega_0) + \beta_3' (\omega_0) \Delta \omega$ and $\omega_0 = 2\pi c / \lambda_0$. We also find that for $N' > 1$, ~50% of the input pulse energy is contained in the soliton over a length of $z_s = 10 \cdot L_{NL}$ [24]. Using Eq. (1) and Eq. (2) we therefore derive the empirical formula...
\[
\Delta \omega = -\frac{1}{\beta_3} \left[ \frac{\tau_0^2 \gamma \hat{P}}{2 \cdot 1.5^2} + \text{sgn}(\beta_3) \beta_2 \right]
\]  \hfill (4)

where \( \beta_{2,3} = \beta_{2,3}(\omega_0) \) and the factor 2 in the denominator results from the fact that only 50% of the incident energy is concentrated in the soliton. As observed in [22], the shift is a linear function of \( \hat{P} \). Since \( \beta_2 \) is presumed to be small, the sign of \( \Delta \omega \) is determined entirely by the sign of \( \beta_3 \), regardless of whether \( \beta_2(\omega_0) \) is normal or anomalous. Figure 8(a) shows a comparison between Eq. (4), the measured data points and NLSE simulations.

![Graphs showing soliton frequency shift and phase difference](image)

Fig. 8. (a) Black line: Expected soliton frequency shift after Eq. (4). Blue line: Extracted from the NLSE simulations. Red dots: Measured. (b) Phase difference between a soliton centered at \( \lambda_s = 783 \) nm and radiation at wavelength \( \lambda \) and corresponding measured spectrum. Dotted line: Phase difference without including the nonlinear term.

Equation (4) does not account for frequency shifts due to intrapulse Raman gain. For moderate fiber lengths (~10 \( L_{NL} \), or 260 mm for a 30 mW input), the Raman effect is small and we find minimal changes in our results if it is excluded from the simulations. For longer lengths (~14 \( L_{NL} \)), the Raman effect becomes significant and we observe larger frequency shifts than those predicted by Eq. (4). For example, for \( L_{NL} = 26 \) mm, Eq. (4) predicts a \( \beta_3 \)-induced shift of 8 nm, which is what we measure for propagation distances shorter than 260 mm. At 350 mm, the shift has increased to 9 nm. In our simulations, we can reproduce this additional shift only when the Raman term is included in the model. When \( \beta_3 \) is negative, Eq. (4) is still valid, though in this case the Raman effect counteracts the frequency shift [7]. Note that the Raman self-frequency shift of the soliton does not change the position of the spectral peak of the non-solitonic radiation (NSR), in contrast to the soliton shift induced by \( \beta_3 \). In the latter case, energy lost by the red-shifted soliton is directly converted into blue-shifted radiation and so drives the NSR spectral recoil, whereas the Raman shift causes energy to be lost to the medium and is an independent process in this case.

As mentioned above, the red shifted soliton loses energy by radiation of a blue shifted dispersive wave. Analytical studies have shown that for soliton propagation perturbed by higher order dispersion, one can find resonance conditions at which the soliton is phase-matched to the NSR [26]. The phase difference between the soliton (frequency \( \omega_s \), propagation constant \( \beta_s \), group velocity \( v_s \)) and the dispersive wave (frequency \( \omega_{NSR} \), propagation constant \( \beta_{NSR} \)) is [29]

\[
\Delta \Phi = \beta_{NSR} - \beta_s - \frac{\left( \omega_{NSR} - \omega_s \right)}{v_s} - \gamma \cdot \hat{P}.
\]  \hfill (5)

#6604 - $15.00 US  
Received 18 February 2005; revised 1 April 2005; accepted 2 April 2005  
(C) 2005 OSA  
18 April 2005 / Vol. 13, No. 8 / OPTICS EXPRESS 2986
If the propagation constants are expanded in a Taylor series around $\omega$, (note that for the numerical NLSE simulations, $\beta$ is expanded around the center frequency of the laser, which gives slightly different values for the $\beta_n$ terms, although the same dispersion relation is described), this equation simplifies to

$$\Delta \Phi = \sum_{n=2}^{\infty} \frac{1}{n!} \beta_n (\omega_{NSR} - \omega_s)^n - \gamma \cdot \hat{P}.$$  

(6)

Figure 8(b) plots $\Delta \Phi$ as a function of wavelength as well as the measured spectrum for $L=250$ mm, $\hat{P}=2400$ W, with $\Delta \Phi=0$ exactly corresponding to the NSR peak. For comparison, we also plot $\Delta \Phi$ without the nonlinear phase term to emphasize its relative importance.

5. Conclusion

We have presented a detailed analysis of femtosecond pulse propagation near $\lambda_{ZD}$ of an ARROW-PCF. Both the linear and nonlinear properties of this fiber are in agreement with numerical simulations. The soliton-like and dispersive wave-like nature of the two frequency components of the pulse have been measured using the FROG technique and behave as expected from theory. We have measured the pulse evolution as a function of both fiber length and input power.

Acknowledgments

This work was produced with the assistance of the Australian Research Council under the ARC Centres of Excellence program. CUDOS (the Centre for Ultrahigh bandwidth Devices for Optical Systems) is an ARC Centre of Excellence.