A Note on Autocorrelations in Asset Returns Due to Market Overreaction

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Abstract

Using the partial price adjustment model of Amihud and Mendelson (1987), it is shown that overreaction in asset returns will produce positive and negative autocorrelations. Market overreaction research use higher order negative correlations to detect market overreaction or trading noise. A proper procedure to detect market overreaction is to use tests for alternated correlation patterns.

Keywords: Autocorrelation, overreaction, price adjustment, trading noise

1 Introduction

An important study in the area of information processing in financial markets and stock return volatility is by French and Roll (1986) (FR)1. FR show that asset prices are much more volatile during exchange trading hours than during non-trading hours. They suggest that the high trading-time volatility is partly attributable to the mispricing of securities in a trading environment where traders may overreact to each other’s trades. This explanation, which they refer to as the ‘trading noise’ hypothesis, has come to be known as the ‘market overreaction’ hypothesis in later studies of information-volatility issues2. Under this hypothesis, FR assert that “unless the market prices are unrelated to the objective economic value of the stock, pricing errors must be corrected in the long run. These corrections would generate negative autocorrelation”3. It is argued that overreaction will lead to negative autocorrelations in security returns up to some arbitrarily specified lag. Consistent with this hypothesis, FR report that daily stock returns are negatively autocorrelated for lags two through 13. Following FR’s result, it has become a standard procedure in the literature to examine market overreaction through serial correlations in returns. For example, Skinner (1989)4 and Kaul and Nirmalendran (1990) (KN)5 compute serial correlations to detect market overreaction in stock returns.

The purpose of this note is to point out that autocorrelations beyond lag one can be positive even in the presence of trading noise or market overreaction. Using the partial price adjustment model of Amihud and Mendelson (1987)6, we show that higher order autocorrelations beyond lag one are not necessarily negative and can be positive, even in the presence of overreaction. In other words, market overreaction in stock returns does not necessarily lead to negative autocorrelations at all lags.

The implications of the results reported in this note are important in two respects. First, it shows that higher-order negative serial correlation is an inappropriate procedure to test for overreaction in stock returns. Positive autocorrelations at some lags are consistent with market overreaction. Second, it explains an important empirical inconsistency in earlier research. KN provide evidence, in contrast to the negative autocorrelation evidence of NYSE returns of FR, that NASDAQ returns are only negatively autocorrelated at lag one, and not thereafter. They conclude that bid-ask errors are the predominant source of negative autocorrelations in stock returns and comment on FR’s result that “The negative autocorrelations up to lag 13 in the returns of NYSE and AMEX firms remain a puzzle”7. The explanation for this puzzle may be that the sources of negative autocorrelation in NYSE/AMEX may be related to other ‘noise’ factors (such as inventory imbalance, random order arrival and nonindependence of the bid-ask bounce) and may not be due market overreaction8. The analysis in this note shows that negative autocorrelation in all higher order lags is not a necessary condition to detect market overreaction. Market overreaction can unambiguously produce positive autocorrelation at some lags.

The next and final section shows how the autocorrelation can be positive even when there is market overreaction. The necessary mathematical proofs are contained in the appendix.

2 Positive Autocorrelation And Overreaction

Amihud and Mendelson (1987) develop a simple model for price behavior in the stock market when prices do not always fully adjust to new information2. This model characterizes $P_t$, the price of the stock observed at time $t$, as a linear combination in $g$ of $P_{t-1}$ and the true value at time $t$, $V_t$:

$$P_t = gV_t + (1-g)P_{t-1} + w$$  \hspace{1cm} (1)

$P_t$ and $V_t$ are in logarithms. $V_t$ follows a random walk with a drift, and $V_tV_{t-1}$, the value return, has a variance of
\(v\)', which is called the intrinsic variance or the ‘true’ variance of the return process. ‘\(g\)' is the price adjustment coefficient with values between, but not including, zero and two. The coefficient \(g\) reflects the adjustment of observed prices toward the security’s true value \(V_t\). A value of \(g < 1\) would indicate partial price adjustment where prices do not fully adjust to new information. When \(g > 1\), we have overshooting or overreaction of traders to new information; a conjecture similar to that of FR. In an efficient market, we should observe the stock price to possess complete information adjustment with \(g=1\).

The term \(u_t\), which has zero mean and finite variance, \(\sigma^2\), is the source of the noise in this process. \(\sigma^2\) is primarily determined by factors such as the liquidity demanded by investors and traders, discreteness of stock prices and the price fluctuation between the bid and the ask. It is important to recognize here that the noise or excess variance generated due to market overreaction \((g > 1)\) is separate from the noise term \(\sigma^2\). For simplicity, as in KN (1990), let us assume that the bid-ask error is the dominant source of noise in observed returns and designate \(\sigma^2\) as the bid-ask induced variance. Therefore, the return generating process will contain some noise due to the bid-ask spread, even when prices fully adjust in an otherwise efficient market, i.e. \(g=1\).

In general, the observed variance, \(\text{Var}(R_t)\), is

\[
\text{Var}(R_t) = \text{Var}(P_t - P_{t-1}) = \frac{gv^2 + 2\sigma^2}{(2-g)}
\]

and, the \(n\)th-order autocorrelation of the observed returns is given as

\[
\text{Corr}(R_t, R_{t-n}) = \frac{v^2 - \frac{\sigma^2}{(1-g)}}{v^2 + \frac{2\sigma^2}{g}}(1-g)^n
\]

Equations (2) and (3) suggest that the magnitude of observed variance and the sign of the autocorrelations are determined by the noise due to bid-ask spread (\(\sigma^2\)) and the price adjustment coefficient (\(g\)). A negative sign on the autocorrelation would indicate that the observed returns contain errors due to the bid-ask spread and/or overreaction by traders. Empirical tests of market overreaction typically rely on the sign of the autocorrelation to detect the presence of overreaction either (i) by removing the bid-ask error (as in KN) or (ii) by assuming the bid-ask errors to be independent from day to day (as in FR). The advantage of equation (3) is that it allows us to separately examine the direction of the bias imparted by the bid-ask spread and the overreaction of traders. If we assume that noise in the return generating process is entirely driven by traders’ overreaction (\(\sigma^2=0\) and \(g=1\)), the sign of the autocorrelation is not always negative. Specifically, starting with a negative first-order coefficient, the autocorrelation will alternate between negative and positive signs at subsequent lags. Therefore, presence of positive autocorrelations in returns purged of noise such as bid-ask errors, does not rule out the presence of market overreaction. In fact, the autocorrelation will be unambiguously positive at even lags.

Why do autocorrelations alternate in sign? In explanation, consider the daily returns on an asset over a period of three days in an environment with some overreaction. If we assume that the observed returns contain errors due to mispricing and the overreaction occurs every day, then day-1’s observed return would be negatively correlated with day-two’s observed return. This negative autocorrelation is the result of traders reacting to day-1’s trading activity in a contrarian manner. Similarly, the return on day-three will be negatively correlated with day-two’s return due to the pricing errors. However, the returns on day-1 and day-three are positively correlated since returns on both days are negatively correlated with day-two’s return. This behavior of the observed returns is generated due to the bias of overreactions which adjusts the daily returns in opposite direction each day.

Throughout the above analysis a key question remains: Is the model of Amihud and Mendelson, as employed here, consistent with the general models of market overreaction? To examine this question, consider the variability of a k-period return in relation to the one-period variance in equation (2). Under the hypothesis that the market fully adjusts to new information \((g=1)\), the value return variance \(v^2\) increases \(k\) times while the noise variance \(2\) remains unchanged. Therefore, with a sufficiently large \(k\), the k-period observed variance has a very small \(\sigma^2\) component and is close to \(k\) times the one-period variance. FR and KN use the same logic, otherwise known as variance ratio tests, to test the null hypothesis that two-day holiday returns are twice the variance of a normal trading day, weekly variances are seven times daily variances, and so forth. Alternatively, if there is overreaction \((i.e., g > 1)\), the overall observed return variance becomes greater than the true return variance; an implication consistent with the overreaction hypothesis of FR and KN. Therefore, the price adjustment model of Amihud and Mendelson is consistent with the predictions of market overreaction or trading noise models employed by FR.

In summary, autocorrelations in stock returns can be positive in the presence of market overreaction or trading noise.
Appendix

The derivation of the results is an extension of the price adjustment model by Amihud and Mendelson (1987). We start with equation (3.1) of their paper which is equation (1) in the text.

The observed price of the asset, \( P_t \), is given as

\[
P_t = g V_t + (1 - g) P_{t-1} + u_t.
\]

Where, ‘\( g \)’ is the price adjustment coefficient, \( V_t \) is the intrinsic value of the security at time \( t \). \( u_t \) is a ‘white noise’ with

\[
E(u_t) = 0, \text{ and } \text{Var}(u_t) = \sigma^2.
\]

The intrinsic value if the asset, \( V_t \), follows a random walk with drift.

\[
V_t = V_{t-1} + e_t + m
\]

where,

Expected ‘value’ return \( = E(V_t - V_{t-1}) = m \)

and \( e_t \)’s are random variables, with

\[
E(e_t) = 0, \text{ Var}(e_t) = \nu^2, \text{ and } \text{Cov}(e_t, e_{t-1}) = \text{Cov}(e_t, u_t) = 0
\]

By induction, it can be shown that

\[
P_t = g \sum_{i=0}^{\infty} (1 - g)^i V_{t-i} + \sum_{i=0}^{\infty} (1 - g)^i u_{t-i}
\]

This is equivalent to equation (3.3) of Amihud and Mendelson. Similarly, the observed return \( R_t \) is given as

\[
R_t = P_t - P_{t-1} = m + g \sum_{i=0}^{\infty} (1 - g)^i (e_i - u_{t-i}) + u_t.
\]

The variance of \( R_t \) is

\[
\text{Var}(R_t) = \frac{g \nu^2}{2 - g} + \frac{2 \sigma^2}{2 - g}
\]

(2)

The nth order correlation coefficient of \( R_t \) is

\[
\text{Cov}(R_t, R_{t-n}) = \text{Var}(R_t) \text{Var}(R_{t-n})^{1/2}
\]

\[
\text{Cov}(R_t, R_{t-n}) = \text{Cov}(R_t, R_{t-n}) / \text{Var}(R_t)
\]

The covariance between \( R_t \) and \( R_{t-n} \)

Using expressions for \( \text{Var}(R_t) \) and \( \text{Cov}(R_t, R_{t-n}) \) we obtain

\[
\text{Corr} (R_t, R_{t-n}) = \frac{\nu^2 - \sigma^2 / (1 - g)^n (1 - g)^n}{\nu^2 + 2 \sigma^2 / g}
\]

(3)

References

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3. Ibid, p.15.


8. Since FR examine NYSE/AMEX stocks and Kaul and Nirmalendran examine NASDAQ stocks, the difference in the structure of autocorrelation may be due to structural differences between the two trading mechanisms and/or due other noise factors. However, as Kaul and Nirmalendran point out, the second explanation can not be tested until sufficient bid-ask data are available for NYSE/AMEX returns.

9. A similar model with lagged price adjustment is presented by Hasbrouck and Ho (1987). See Hasbrouck, Joel and Thomas S.Y. Ho, “Order Arrival, Quote Behaviour and the Return Generating Process”, (1987), Journal of Finance, 42. Hasbrouck and Ho characterize the bid-ask error as a binomially distributed component of the observed return; separate from other ‘noise’ factors which are normally distributed with zero mean. Amihud and Mendelson’s model is employed here since it subsumes the bid-ask spread within the error component of the return generating process. This allows a clearer depiction of the interaction between the ‘true’ variance and the noise variance. However, in the limiting case of a pure random walk or Roll’s (1984) bid-ask error model, both give exactly same analytics. See Roll, Richard, “A Simple Implicit Measure of the Bid-Ask Spread in an Efficient Market” (1984), Journal of Finance, 39. Hasbrouck and Ho do not specifically consider market overreaction; but their model can be generalized to include overreaction and to obtain the same results as in here.

10. In principle, this is similar to the Kaul and Nirmalendrars’s procedure of removing the bid-ask bounce from observed
returns by constructing bid-to-bid returns. However, note that there may be some other noise components that remain in the bid-to-bid returns. These noise sources could be random order arrival or temporary inventory imbalance.